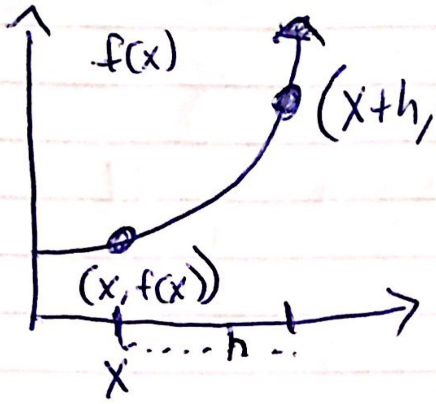
UNIT 3

Definition of Derivative



Difference Quotient

$$\boxed{m_{sec} = \frac{f(x+h) - f(x)}{h}}$$

$$\boxed{m_{tan} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

Definition of Derivative

Ex 1) Find the derivative by using the definition of derivative

Ex 1) $f(x) = 2x+3$

$$\lim_{h \to 0} \frac{2(x+h) + 3 - [2x+3]}{h}$$

$$\lim_{h \to 0} \frac{2x+2h+3-2x-3}{h}$$

$$\lim_{h \to 0} \frac{2h}{h} = \lim_{h \to 0} 2 = \boxed{2}$$

Ex 2) Find the derivative using the definition

$f(x) = x^2 + 2$

Ex 2) $\lim_{h \to 0} \frac{(x+h)^2 + 2 - [x^2+2]}{h}$

$\lim_{h \to 0} \frac{x^2 + 2xh + h^2 - x^2 - 2}{h}$

$\lim_{h \to 0} \frac{2xh + h^2}{h} = \lim_{h \to 0} 2x + h = \boxed{2x}$

Ex 3) Find the derivative of $y = 3x^2 + x$ by using the definition

$\lim_{h \to 0} \frac{3(x+h)^2 + (x+h) - [3x^2 + x]}{h}$

$\lim_{h \to 0} \frac{3(x^2 + 2xh + h^2) + x + h - 3x^2 - x}{h}$

$\lim_{h \to 0} \frac{3x^2 + 6xh + 3h^2 + x + h - 3x^2 - x}{h}$

$\lim_{h \to 0} \frac{6xh + 3h^2 + h}{h} = \lim_{h \to 0} 6x + 3h + 1 = \boxed{6x + 1}$

Product and Quotient Rule

Product Rule

$y = f(x)g(x)$

$y' = f(x)g'(x) + g(x)f'(x)$

Ex 1) $y = (x^2+2)(4x-3)$

$y' = (x^2+2)(4) + (4x-3)(2x)$

Ex 2) $y = (3x-5)4g(x)$

$y' = (3x-5)4g'(x) + 4g(x)(3)$

Ex 3.) $y = (x^2+3x)(5x-2)$

$y' = (x^2+3x)(5) + (5x-2)(2x+3)$

Quotient Rule $y = \dfrac{f(x)}{g(x)}$ then $y' = \dfrac{g(x)f'(x) - [f(x)g'(x)]}{[g(x)]^2}$

Ex 1) $y = \dfrac{2x-3}{4x+6}$

$$y' = \frac{(4x+6)(2) - [(2x-3)4]}{(4x+6)^2}$$

$$y' = \frac{8x+12-8x+12}{(4x+6)^2} = \frac{24}{(4x+6)^2}$$

Ex 2.) Find $\frac{dy}{dx}$ (derivative)

$$y = \frac{4x^4 - 6x^3 + 3x^2 + x}{x}$$

Rewrite $y = 4x^3 - 6x^2 + 3x + 1$

$$y' \text{ or } \frac{dy}{dx} = 12x^2 - 12x + 3$$

Ex 3) Find y' $\quad y = \frac{2x}{5x-3}$

$$y' = \frac{(5x-3)(2) - [2x(5)]}{(5x-3)^2} = \frac{-6}{(5x-3)^2}$$

Chain rule

$y = f(g(x))$ then $y' = f'(g(x))\,g'(x)$

" Derivative of outside · derivative of inside "

Recall

$y = 2x^3$

$y' = 6x^2$

Ex 1 Chain

$y = 2(3x-2)^3$

$y' = 6(3x-2)^2 \cdot 3$

$y' = 18(3x-2)^2$

$y = -x^4$

$y' = -4x^3$

Ex 2

$y = -(x^2+3)^4$

$y' = -4(x^2+3)^3 \cdot 2x$

$y' = -8x(x^2+3)^3$

Recall

$y = -4x^3$

$y' = -12x^2$

Ex 3 $y = -4(5x-3)^3$

$y' = -12(5x-3)^2 \cdot 5$

$\boxed{y' = -60(5x-3)^2}$

* Note the chain rule is used with EVERY derivative, the derivative of the inside is typically 1.

so $y = 4(x)^3 \quad y' = 12x^2 \cdot 1 = 12x^2$

Derivative of Trig Functions

$y = \sin x$
$y' = \cos x$

$y = \cos x$
$y' = -\sin x$

$y = \sec x$
$y' = \sec x \tan x$

$y = \csc x$
$y' = -\csc x \cot x$

$y = \tan x$
$y' = \sec^2 x$

$y = \cot x$
$y' = -\csc^2 x$

Ex 1) Find the slope of the tangent line of $y = \sin x$ at $x = \pi/4$

$f(x) = \sin x$
$f'(x) = \cos x$

$f'(\pi/4) = \cos(\pi/4) = \sqrt{2}/2$

$\boxed{m = \frac{\sqrt{2}}{2}}$

Ex 2 $f(x) = 4x\cos x$ Find $f'(\pi/3)$

product rule

$f'(x) = -4x\sin x + 4\cos x$

$f'(\pi/3) = -4\pi/3\,(-\sqrt{3}/2) + 4(1/2) = \boxed{\frac{-2\sqrt{3}}{3} + 2}$

Ex 3) given $f(x) = \frac{4(x^2-2)^5}{\cos x}$

find $f'(x)$

* chain
* quotient
* trig

$$f'(x) = \frac{\cos x\left[20(x^2-2)^4(2x)\right] - \left[4(x^2-2)^5(-\sin x)\right]}{\cos^2 x}$$

Base e, base a, ln, and log base a

Base e

$y = e^u$
$y' = e^u \cdot u'$

Ex 1 $y = e^{3x}$
$y' = 3e^{3x}$

Ex 2
$y = 4e^{-2x}$
$y' = -8e^{-2x}$

Base a

$y = a^u$
$y' = a^u \cdot u' \cdot \ln(a)$

Ex 1 $y = 2^x$
$y' = 2^x \cdot 1 \cdot \ln 2 = 2^x \ln(2)$

Ex 2
$y = 3^{5x}$
$y' = 3^{5x} \cdot 5 \cdot \ln(3)$

ln Rule

$y = \ln u$ $y' = \frac{u'}{u}$

Ex 1) $y = \ln(5x)$ $y' = \frac{5}{5x} = \frac{1}{x}$

Ex 2) $y = 2\ln\left(\frac{2}{3}x\right)$

$y' = 2 \cdot \frac{\frac{2}{3}}{\frac{2}{3}x} = \boxed{\frac{2}{x}}$

log a rule

$y = \log_a u$ $y_1 = \frac{u'}{u\ln(a)}$

Ex 1 $y = \log_3(5x)$

$y' = \frac{5}{5x\ln(3)}$

$y' = \boxed{\frac{1}{x\ln(3)}}$

Implicit differentiation

Explicit form	Implicit form
$y = 2x + 3$	$2x - 3y = 5$
$y = 4x^2 - 3x + 5$	$x^2 + 2xy - 3x = 4x - y^3$
$y = 2x\cos x$	$x - y^3 - 3y = 2y^2 + 4x - 5$

Steps for solving Implicit derivatives

- Take the derivative of each term
- When taking the derivative of a y term, put $\frac{dy}{dx}$ behind it.
- Solve for $\frac{dy}{dx}$

* ALGEBRA
* $\frac{dy}{dx}$ terms left / bottom
* non $\frac{dy}{dx}$ terms right / top

Ex 1) Find $\frac{dy}{dx}$

$$2x^2 - 3y^3 = 2x - 3$$

$$4x - 9y^2\frac{dy}{dx} = 2$$

$$\boxed{\frac{dy}{dx} = \frac{2 - 4x}{-9y^2}}$$

Ex 2) Find $\frac{dy}{dx}$ at the point $(1, 2)$

for $4y^2 - 3x - 2y = 4x^2 - 3$

$$8y\frac{dy}{dx} - 3 - 2\frac{dy}{dx} = 8x$$

$$\frac{dy}{dx} = \frac{8x + 3}{8y - 2} \quad @(1,2) = \boxed{\frac{11}{14}}$$

Ex 3) Find $\frac{dy}{dx}$ for $2xy - 3y^2 = 2x + 4$

product rule

$$2x\frac{dy}{dx} + 2y - 6y\frac{dy}{dx} = 2$$

$$\boxed{\frac{dy}{dx} = \frac{2-2y}{2x-6y} \quad \text{or} \quad \frac{1-y}{x-3y}}$$

Equation of Tangent line

Point slope form $y - y_1 = m(x - x_1)$

We use $y - \_\_\_ = \_\_\_ (x - \_\_\_)$

(x, y) x in blank 3
y in blank 1
$f'(x)$ in blank 2

Ex) Write the equation of the tangent line to $f(x) = x^2 + 2x + 1$ at the point where $x = 1$

- So we have $x = 1$ so $y = (1)^2 + 2(1) + 1 = 4 = y$

- To get slope at $(1, 4)$ we find $f'(x)$ and plug in x

$f'(x) = 2x + 2$ $\quad f'(1) = 2(1) + 2 = 4$

so $\boxed{y - \underline{4} = \underline{4}(x - \underline{1})}$

Calculator Instructions for Equation of tangent line

Ex: Write the equation of the line tangent to $f(x) = x^2 + 2x + 1$ at $x = 1$

Go to $y_1 =$ type in $x^2 + 2x + 1$

$y_2 = \boxed{\text{math} \mid 8} \quad \frac{d}{d\boxed{x}}\left(\boxed{Y_1}\right)\Big|_{\boxed{x} = \boxed{x}}$

To get y_1 $\boxed{\text{Vars}}$ $\boxed{\text{Y-Vars}}$ $\boxed{\text{Function}}$ $\boxed{Y_1}$

Go to $\boxed{\text{Table}}$

X	Y_1	Y_2
1	4	4
blank 3	blank 1	bank 2

so $y - 4 = 4(x-1)$

or

$y - 4 = 4x - 4$ or In slope intercept form $y = 4x$ $\quad y = mx + b$

Higher order derivatives

$y'' = \frac{d^2y}{dx^2} = f''(x)$ = all mean take derivative twice

Ex $y = 5x^3 - 3x^2 + 2x + 1$ $\quad y' = 15x^2 - 6x + 2$

$y'' = 30x - 6$

$y''' = 30$

Related Rates

Steps

1. Draw a picture
2. Write the formula
3. Write each variable in formula
4. Write the rate for each variable in formula
5. Identify what is given and what is asked for.
6. Find the derivative of the formula with respect to (wrt) time. - [implicit differentiation]
7. Plug in what is given and solve for what is asked for
8. Identify units for answers - Volume ^{3}
   - Area ^{2}
   - All else ^{1}
9. Interpret answer to be sure it makes sense

Ex of derivative of formula wrt time

① $V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

② $a^2 + b^2 = c^2$

$2a\frac{da}{dt} + 2b\frac{db}{dt} = 2c\frac{dc}{dt}$

Scanned by CamScanner

AP FRQ Reference for unit 3

AB

2017 1d 2b 4ab 6abc
2016 4b 5c 6ab
2015 4b 6abc
2014 1ad 3cd 4d 5c 6b
2013 1a 3d 4d 6a
2012 4ab 5b
2011 Form A 3a 5ab 6b
2011 Form B 2c 3c
2010 Form A 6ab
2010 Form B 2cd 3d

* Note you can research more years on your own Note there have been more FRQs involving UNIT 3 in the recent years so the trend is completely Knowing rules (derivative), equations of tan lines and related rates. MC question also are involved.