UNIT 5A

Particle motion $x(t)$ or $y(t)$

position $= s(t)$

velocity $= s'(t)$ or $v(t)$

acceleration $= s''(t)$ or $v'(t)$ or $a(t)$

speed $= |v(t)|$

$x(t) \longleftrightarrow$ left/right movement

increasing = right
decreasing = left
change direction = max/min

$y(t) \updownarrow$

increasing = up
decreasing = down
change direction = max/min

speed <u>increasing</u> if velocity and acceleration have same sign

speed <u>decreasing</u> if velocity and acceleration have opposite sides.

Alternative form of a derivative

Some functions you cant determine derivative at a point with the derivative rules,

(like absolute value functions)

So we check to see if slopes on both sides match

$$\boxed{\lim_{x \to c} \frac{f(x) - f(c)}{x - c}}$$

Example $f(x) = |x-3|$ find $f'(3)$

no rule so use

$$\lim_{x \to 3} \frac{|x-3| - 0}{x - 3}$$ so $$\lim_{x \to 3} \frac{|x-3|}{x-3}$$

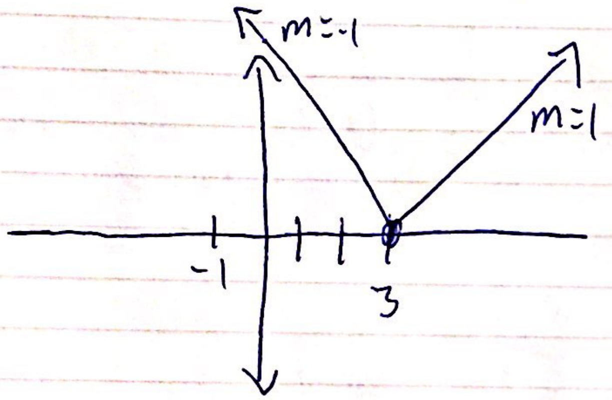
Left side

$$\lim_{x \to 3^-} \frac{|x-3|}{x-3} = -1$$ $$\lim_{x \to 3^+} \frac{|x-3|}{x-3} = 1$$

"create a table to see"

lim from left $\neq$ lim from right so DNE

Think about the picture/graph $y = |x-3|$



slope of tan line to right of 3 is +1

slope of tan line to left of 3 is -1

Ex 2 $f(x) = |x-3|$ find $f'(-1)$

By looking at graph above the left and right slope of $x=-1$ is -1

so $f'(-1) = -1$

L'Hopital's Rule for Limits

When plugging in c for a limit, if answer results in $\frac{0}{0}$ or $\frac{\infty}{\infty}$ you can use L'Hopital's Rule

If $\lim\limits_{x \to c} \frac{f(x)}{g(x)} = \frac{0}{0}$ or $\frac{\infty}{\infty}$

then $\lim\limits_{x \to c} \frac{f'(x)}{g'(x)}$ can be applied.

Ex) $\lim\limits_{x \to 0} \frac{\sin x}{x} = \frac{\sin(0)}{0} = \frac{0}{0}$

Apply Rule $\lim\limits_{x \to 0} \frac{\cos x}{1} = \cos(0) = \boxed{1}$

Ex) $\lim\limits_{x \to 0} \frac{1 - \cos x}{x} = \frac{1 - \cos(0)}{(0)} = \frac{1-1}{0} = \frac{0}{0}$

Apply Rule $\lim\limits_{x \to 0} \frac{0 + \sin x}{1} = \sin x = \sin(0) = \boxed{0}$

Log Differentiation

You can apply log properties to simplify the process of finding derivatives

Use when the exponent is a variable Log Both sides

$$\boxed{\begin{aligned} \ln ab &= \ln a + \ln b \\ \ln \frac{a}{b} &= \ln a - \ln b \\ \ln a^n &= n \ln a \end{aligned}}$$

Remember

$y = \ln u \quad y' = \frac{u'}{u}$

Ex 1) Find $\frac{dy}{dx}$ for $y = \ln\sqrt{x^2+4}$

so $y = \frac{1}{2}\ln(x^2+4)$

$$\frac{dy}{dx} = \frac{1}{2} \cdot \frac{2x}{x^2+4} = \boxed{\frac{x}{x^2+4} = \frac{dy}{dx}}$$

Ex 2)

$y = x^x$ find $\frac{dy}{dx}$

$\ln y = \ln x^x$ so $\ln y = x \ln x$

$$\frac{1}{y}\frac{dy}{dx} = x \cdot \frac{1}{x} + \ln x$$

$$= 1 + \ln x$$

$$\frac{dy}{dx} = y(1+\ln x)$$

$= x^x(1+\ln x)$