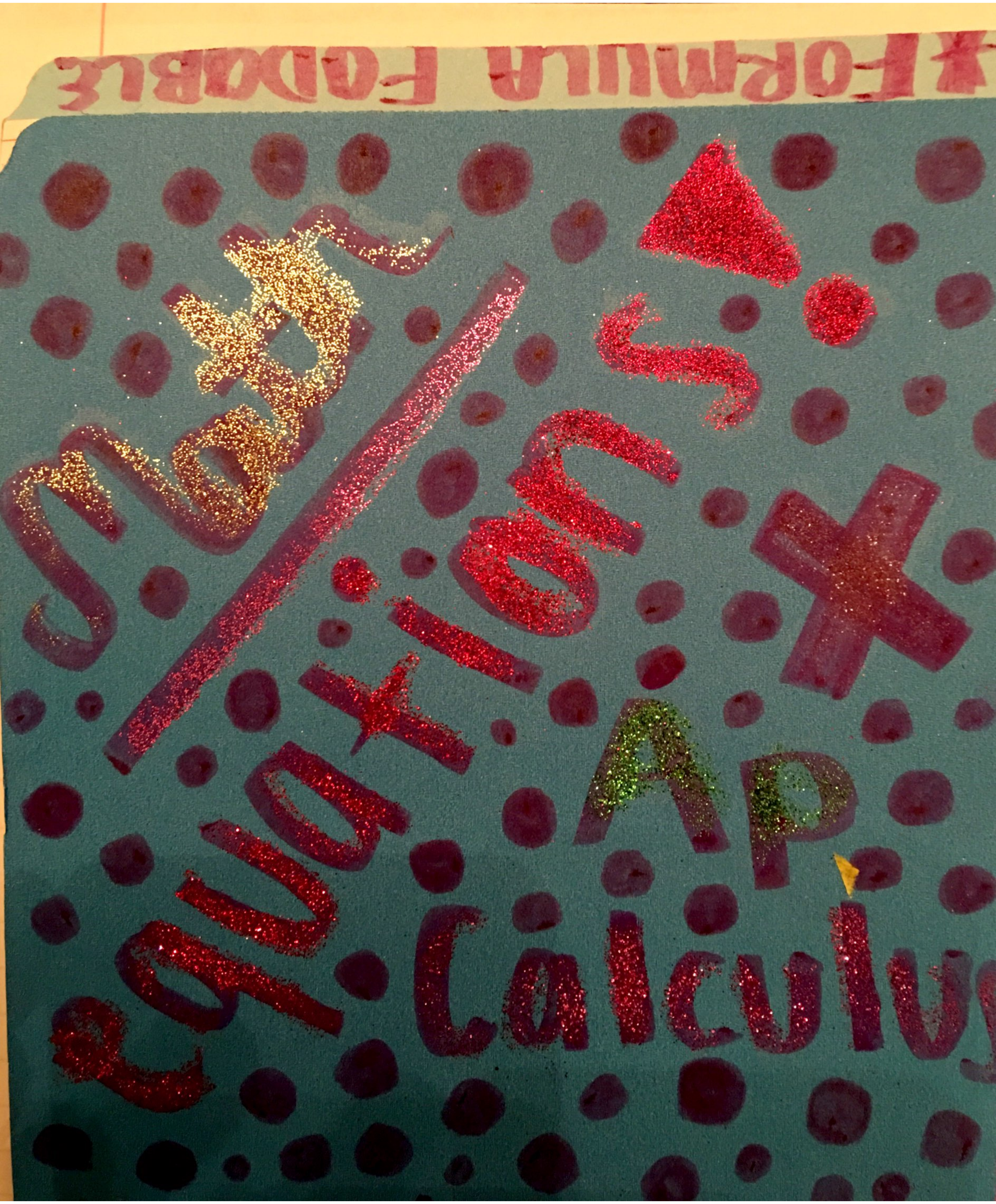

Math
Equations
AP
Calculus

Big Card 1

Derivative Rules

Chain Rule

$y = a(u)^n$

$y' = an(u)^{n-1}u'$

Power Rule

$y = ax^n$

$y' = anx^{n-1}$

Product Rule

$y = uv$

$y' = uv' + vu'$

Quotient Rule

$y = \frac{u}{v}$

$y' = \frac{vu' - [uv']}{(v)^2}$

Trig

$y = \sin u$

$y' = u'\cos u$

$y = \cos u$

$y' = -u'\sin u$

$y = \sec u$

$y' = u'\sec u \tan u$

$y = \csc u$

$y' = u'\csc u \cot u$

$y = \tan u$

$y' = u'\sec^2 u$

$y = \cot u$

$y' = -u'\csc^2 u$

Inverse Trig

$y = \sin^{-1} u$

$y' = \frac{u'}{\sqrt{1-u^2}}$

$y = \cos^{-1} u$

$y' = \frac{-u'}{\sqrt{1-u^2}}$

$y = \sec^{-1} u$

$y' = \frac{u'}{|u|\sqrt{u^2-1}}$

$y = \csc^{-1} u$

$y' = \frac{-u'}{|u|\sqrt{u^2-1}}$

$y = \tan^{-1} u$

$y' = \frac{u'}{1+u^2}$

$y = \cot u$

$y' = \frac{-u'}{1+u^2}$

Big Card #1 Continued

Derivative Rules

log and exponential

$y = e^u$
$y' = u'e^u$

$y = a^u$
$y' = u'a^u \ln(a)$

$y = \ln u$
$y' = \frac{u'}{u}$

$y = \log_a u$
$y' = \frac{u'}{u \ln(a)}$

Inverse Derivative

$g'(x) = \frac{1}{f'(G(x))}$ g is the inverse function of f

Implicit Differentiation

Ex $2xy - 3y^2 = 4y - 2x$

$2x\frac{dy}{dx} + 2y - 6y\frac{dy}{dx} = 4\frac{dy}{dx} - 2$

$\frac{dy}{dx} = \frac{-2-2y}{2x-6y-4}$ or $\frac{-1-y}{x-3y-2}$

Big Card #2

Integration Rules

Indefinite

$\int f'(x)dx = f(x) + C$

Finding y-values f(b) given (a, f(a)) and f'(x)

$f(b) = f(a) + \int_a^b f'(x)dx$

Ex $f(2) = 300$ $f'(x) = V_a(t)$

Find $f(12)$

$f(12) = 300 + \int_2^{12} V_a(t)dt$

Definite

$\int_a^b f'(x)dx = f(b) - f(a)$

U-Sub shortcut

$\int f'(Kx)dx = \frac{1}{K} f(Kx) + C$ (K is a #)

Derivative of Integral FTOC

$F(x) = \int_3^{5x} 3t^2 - 4t \, dt$ $F'(x) = [3(5x)^2 - 4(5x)] \cdot 5$

Riemman Sum

X	0	5	10	15	20
f(x)	1	2	3	4	5

Left (4 subs)
(5)(1)
(5)(2)
(5)(3)
+ (5)(4)

Right (4 subs)
(5)(2)
(5)(3)
(5)(4)
+ (5)(5)

Midpt (2 subs)
(10)(2)
+ (10)(4)

Trapezoid (4 subs)

$\frac{5}{2}(1 + 2(2) + 2(3) + 2(4) + 5)$

Small Card
#1

Limits

3 methods

Numerical	Graphical	Analytical Rules
y= Tblstart ; c ΔTbl : .001	↳ Finger Test $\lim_{x \to c}$ exist if both fingers come to same y-value	① Plug In, if 0/0 or #/0 - a) factor + cancel - b.) rationalize + cancel - c.) Simplify fraction + cancel

Special Trig Limit

a) $\lim_{x \to 0} \frac{\sin x}{x} = 1$ b.) $\lim_{x \to 0} \frac{1-\cos x}{x} = 0$

Infinite Limits

Deg Top > Deg Bttm = + or − ∞

Deg Top < Deg Bttm = 0

Deg Top = Deg. Bttm = ratio of lead coefficient

Small Card #2

Continuity

Continuous at a point c if

1.) $\lim_{x \to c^-} = \lim_{x \to c^+} = L$

2.) f(c) exist

3.) $\lim_{x \to c} = f(c)$ limit = functional value

Ex Is f(x) continuous at x=2

$$f(x) = \begin{cases} x^2-3, & x<2 \\ 2x-3, & x \geq 2 \end{cases}$$

1) $\lim_{x \to 2^-} (2)^2-3 = 1$

$\lim_{x \to 2^+} (2)(2)-3 = 1$

2) $f(2) = 2(2)-3 = 1$

$\lim_{x \to 2} = f(2) = 1$

YES

Small
Card # 3

Theorems

Let f(x) be continuous on [a,b] and differentiable on (a, b), then....

(MVT) 1) <u>Mean Value Theorem</u> = $f'(c) = \frac{f(b)-f(a)}{b-a}$

(EVT) 2.) <u>Extreme Value Theorem</u> f(x) has both a min and a max on the interval or at the endpoints (✓ the endpoints)

(IVT) 3.) <u>Intermediate Value Theorem</u> - K is a number between f(b) and f(a), then there is a c such that f(c) = K

Small Card
#4

Related Rates

1.) Draw Picture
2.) Determine Formula
3.) Write each variable in formula
4.) Write rate for each variable in formula wrt time
5.) Take derivative of formula wrt time
6.) Plug in what you know and solve for what you are trying to find
7.) Place correct units

All else = units
Area = $units^2$
Volume = $units^3$

Ex $A = \pi r^2$

$A = 4\pi$
$r = 2$
$\frac{dA}{dt} = ?$
$\frac{dr}{dt} = 3$

so $\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$? = 2(\pi)(2)(3)$

$\frac{dA}{dt} = 12\pi \ units^2$

Small card
#5

Extrema + Concavity

f(x)	↗	↘	↗
f'(x)	+	−	+

(inc/dec) (max/min) of f(x)

f'(x)	↗	↘	↗
f''(x)	+	−	+

(concavity / Inf pts) of f(x)

Ex: To find max/min points and intervals of incr/decr using 1st derivative test

$f(x) = 4x^2 - 2x$

$f'(x) = 8x - 2 = 0$

$x = 1/4$

− | +

$f'(0)$ $\frac{1}{4}$ $f'(1)$

∴ min point is $(1/4, f(1/4))$

decreasing $(-\infty, 1/4)$

increasing $(1/4, \infty)$

You may also use the 2nd derivative test

$f''(x) = 8$ $f''(1/4) = +$ ∪ so min

Find inflection points and intervals of concavity

$f(x) = x^3 - 3x^2$

$f'(x) = 3x^2 - 6x$

$f''(x) = 6x - 6 = 0$

$x = 1$

− | +

$f''(0)$ 1 $f''(2)$

inflection pt $(1, f(1))$

concave down $(-\infty, 1)$

concave up $(1, \infty)$

Small
card #6

Differential Equations

5 pts on AP Exam

1. Seperate Variables (1)
2. Integrate both sides (2)
   look for $\int \frac{u'}{u} = \ln|u| + c$

3. Plug In initial condition (x, y) and solve for C (1)
4. Place the c you found back into function and solve for y (1)

if given other variables
like $\frac{dw}{dt} = t^3(100-w)$
change to $\frac{dy}{dx} = x^3(100-y)$

small card #7

Calculator Knowledge

Derivative (graph or table) math 8

$y_1 = f(x)$

$y_2 = \frac{d}{d\boxed{x}}[y_1]_{x=x}$

$y_3 = \frac{d}{d\boxed{x}}[y_2]_{x=x}$

Definite Integration math 9

$\int_a^b \boxed{Y_1}\, d\boxed{x}$ integrate on home screen

Finding Zeroes

Put function in y=

2nd Trace 2, left bound enter, right bound enter, guess enter

Finding intersection pts (volumes of solids)

Put functions in y_1 and y_2

2nd, Trace, 5, 1st curve enter, 2nd curve enter, enter

To paste y_1 - Vars, Y vars, function

Small card
#8

(a, c) (b, d)

Volumes of Solids

Area $= \int_a^b \text{Top} - \text{Bttm}\, dx$ or $\int_c^d \text{Right} - \text{Left}\, dy$

$f(x) - g(x)$ $\quad f(y) - g(y)$

Revolved Volume = Place pencil on line you revolve about

$\pi \int_a^b \left(\text{outer function}\right)^2 - \left(\text{inner function}\right)^2 dx$

$(R(x))^2 - (r(x))^2$

Volume of Cross Section =

square $\int_a^b (\text{area})^2 dx$

isosceles rt Δ $\frac{1}{2} \int_a^b (\text{area})^2 dx$

semicircle $\frac{\pi}{8} \int_a^b (\text{area})^2 dx$

rectangle $K \int_a^b (\text{area})^2 dx$

small card #9

Miscellaneous - AP Exam Tips

Format
- 30 MC No Calculator (60 min)] 50%
- 15 MC Calculator (45 min)]

10min Break
- 2 FRQ Calculator (30 min)
- 4 FRQ No Calculator (60 min)

Equation of tangent line $y - \_ = \_(x - \_)$

(normal line) → perpendicular to tangent line

Alternative form of derivative

$$\lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$ = can use for abs value

---

Speed = |velocity|

increasing speed = velocity and acceleration have same sign

decreasing speed = velocity and acceleration have different sign

position = original
velocity = first derivative
acceleration = 2nd derivative

Trigonometry

$y = \sin u$ — $y' = u'\cos u$

$y = \cos u$ — $y' = u'\sin u$

$y = \sec u$ — $y' = u'\sec u \tan$

$y = \tan u$ — $y' = u'\sec$

Inverse Trig *

$y = \sin^{-1} u$ — $y' = \frac{u'}{\sqrt{\;}}$

$y = \cos^{-1} u$ — $y' = \frac{-u'}{\sqrt{\;}}$

Integration Rules and Riemann Sums

Integration Rules

Indefinite: $\int f'(x)dx = f(x) + C$

definite: $\int_a^b f'(x)dx = f(b) - f(a)$

u-sub (K is a #) $\int f(kx)\,dx = \frac{1}{k} f'(kx) + C$

Ex. $\int \cos\left(\frac{4\pi}{3}x\right)dx$: $K = \frac{4\pi}{3}$ so $\frac{3}{4\pi}\sin\left(\frac{4\pi}{3}x\right) + C$

Finding y-values: f(b) given coordinates (2, f(2)) and derivative f'(x)

$f(b) = f(a) + \int_a^b f'(x)dx$ Ex $f(2) = 300$ so ?

find f(12) $f(12) = 300 + \int_2^{12} \sqrt{a(x)}\,dx$

$f'(x) = \sqrt{A}\,(x)$

Integra…

Riemann…

x	0	5
f(x)	1	2

Left (4 subintervals): (5)(1) = 5, (5)(2) = 10, (5)(3) = 15, (5)(4) = 20 → 50

midpoint (2 subint): (10)(2) = 20, (10)(4) = 40 → 60

Right (4 subintervals): (5)(2) = 10, (5)(3) = 15, (5)(4) = 20, (5)(5) = 25

$y = 5[3(5x)^2 - 4(5x)]$ — plug in top function for t, then multiply by the derivative of top function

Trapezoid 4 (subint) — short cut (only is x values are equal distances apart)

$\frac{5}{2}(1+2) + \frac{5}{2}(2+3) + \frac{5}{2}(3+4) + \frac{5}{2}(4+5) = 60$

$\frac{5}{2}(1 + 2(2) + 2(3) + 2(4) + 5) = 60$

dx $2x - 6y - 4$ | $x - 3y - 2$ |

chain rule — $y = au^n$, $y' = anu^{n-1} \cdot u'$

*power rule — $y = ax^n$, $y' = anx^{n-1}$

$y = \ln u$ — $y' = \frac{u'}{u}$

$y = \log_a u$ — $y' = \frac{u'}{u\ln(a)}$

quotient rule: $= vu' - [uv'] / (v)^2$

a^u — $a^u \ln(a)$

Related…

Steps

1. draw a picture
2. write formula
3. write each varia… in formula
4. write each rate each variable w/ respect to time
5. Take derivative the formula with time

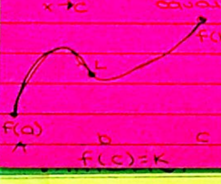
Continuity

Continuous at a point c if:

i) $\lim_{x\to c^-} = \lim_{x\to c^+} = L$

ii) f(c) exist

iii) $\lim_{x\to c} = f(c)$ → 2 side limit equals y-value



Example

is f(x) continuous @ x = 2

$f(x) = \begin{cases} x^2 - 3, & x < 2 \\ 2x - 3, & x \geq 2 \end{cases}$

i) $\lim_{x\to 2^-} = (2)^2 - 3 = 1$, $\lim_{x\to 2^+} 2(2) - 3 = 1$

ii) $f(2) = 2(2) - 3 = 1$

iii) $\lim_{x\to 2} f(x) = f(2) = 1$

Turn off and on [=] vs =

Finding intersection pts

put functions in y, and

FRQ calc (zoom…) slope not… on both sid… one @ x = a

Equation of Tan…

$y - f(x_1) = f'(x_1)$

Mean VT: $f'(c) = f$…

Intermediate VT: K… number between … and f(a) then there is a c where f(c) = K

Rolle's VT: if f(b) = f(a) you are guaranteed a max and min on (a, b)

Special Trig Limits

$\lim_{x\to 0} \frac{\sin x}{x} = 1$

$\lim_{x\to 0} \frac{1 - \cos x}{x} = 0$

Infinite Limits

- deg top > d. bottom: → −∞ or ∞
- deg top < deg bottom: 0
- deg top = deg bottom: ratio of lead coefficient

…$a)^2\,dx$

Rectangle

$x = 1/4$

f'(0) 1/4 f'(1)

concdown $(-\infty, 1)$

concup $(1, \infty)$

dec $(-\infty, 1/4)$ min $(1/4, F(1/4))$

inc $(1/4, \infty)$ infpt $(1, F(1))$

Short cut

2nd D test

$f''(x) = 8$

$f''(1/4) = 8$

min

$\frac{dy}{dx} = x^3(100 - y)$


Derivative Rules
Integration Rules
Indefinite
Definite
u-sub shortcut
Finding y value f(b) given condition (a, f(a)) + derivative f'(x)
Riemman Sum
Left
Right
Midpoint
Trapezoid
Derivative of an Integral
* Don't forget to multiply by the derivative
Shortcut b/c equal intervals
Related Rates
Volumes of Solids
Calculator
Theorems
Mean Value Theorem (MVT)
Differential
Limits
3 methods
Numerical - Table
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Alternative form of
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you got this!!
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Miscellaneous - AP Exam TIPS
Speed = |velocity|
Pendaflex
File Folders