50% of AP Exam

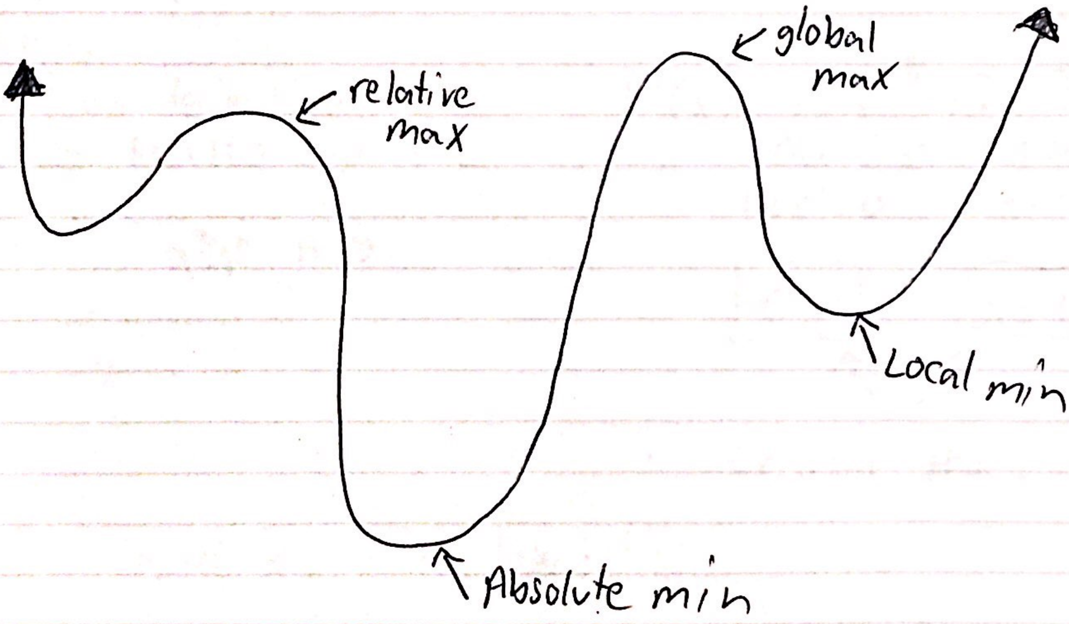
UNIT 4

Extrema

Extrema - refers to max or min points of a function

Relative/Local - max or min in a small space or interval of a function

Absolute/Global - max or min of a entire graph/function
Highest of High or Lowest of Low



Critical #'s - The values obtained by setting derivative equal to 0

- Zeroes of Derivative
- where derivative is undefined

Recall 

max/min happens at horizontal tangent lines $f'(x)=0$

- To find we set derivative (slope) $=0$ and solve to find the critical #'s

Ex 1) Find the critical #'s

$$f(x)=3x^4-4x^3$$

steps

① Find derivative

② set $f'(x)=0$ and solve

$ab=0 \quad a=0 \quad b=0$

$$f'(x)=12x^3-12x^2=0$$

$$12x^2(x-1)=0$$

$$12x^2=0 \qquad x-1=0$$

$$\boxed{x=0} \qquad \boxed{x=1}$$

Critical #'s

Ex 2) Find the critical #'s

$$f(x)=\frac{4x}{x^2+1} \qquad f'(x)=\frac{(x^2+1)(4)-[4x(2x)]}{(x^2+1)^2}$$

$a/b \quad b\neq 0 \quad$ so $\quad (x^2+1)(4)-[4x(2x)]=0$

cont) $4x^2+4-8x^2=0$

$-4x^2+4=0$

$-4(x^2-1)=0$ $\qquad x^2-1=(x-1)(x+1)$

$-4=0 \qquad (x-1)=0 \qquad (x+1)=0$

$\boxed{x=1} \quad \boxed{x=-1}$

critical #'s

Recall since the denominator of $f'(x)$ is $(x^2+1)^2$ there are no restrictions on the domain of $f'(x)$

Ex 3) $y=\frac{1}{2}x-\sin x \quad [0,2\pi]$

$y'=\frac{1}{2}-\cos x=0 \qquad \cos x=\frac{1}{2}$

$\arccos(\frac{1}{2})= \boxed{\frac{\pi}{3}} \quad \boxed{\frac{5\pi}{3}}$

$\boxed{0} \quad \boxed{2\pi}$

critical #'s

Finding Extrema on a Interval

Finding Absolute max/min we need the

Y-VALUES OF THE CRITICAL # AND ENDPOINTS

Ex 4) Find absolute extrema on $[0,6]$

$f(x)=2x^2-8x$

$f'(x)=4x-8=0$

$x=2$

so $f(0)=0 \quad (0,0)$

$f(6)=24 \quad (6,24)$

$f(2)=-8 \quad (2,-8)$

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So minimum value is -8 and it occurs at $x=2$

maximum value is 24 and it occurs at $x=6$

* $\boxed{\text{The Extreme Value Theorem}}$ - if f is continuous on a closed interval $[a,b]$, then f has both a minimum and a maximum on the interval, this can occure where $f'(x)=0$ OR THE ENDPOINTS.

Ex 5) Find the absolute Extrema

$$f(x) = x^3 - 3x^2 + 3x - 2$$

$$f'(x) = 3x^2 - 6x + 3 = 0$$

$$3(x^2 - 2x + 1) = 0$$

$$3(x-1)(x-1) = 0$$

$$x - 1 = 0$$

$$\boxed{x=1}$$

$f(1) = -1$

$(1, -1)$

1^{st} Derivative Test

Purpose - Use the slopes of Tangent lines to find
- max/min pts
- Intervals of Increasing/Decreasing of f(x)

Steps
1) Find the derivative
2.) Set $f'(x) = 0$ and solve for x
3.) pick a # on the left and right side of your critical #'s.
4.) Plug test #'s into the derivative to determine the sign of the slope
5.) Fill out a sign line
6.) Determine Interval of Increasing/Decreasing from your sign line

$-$ to $+$ = decreasing then increasing $\boxed{\text{min}}$

$+$ to $-$ = increasing then decreasing $\boxed{\text{max}}$

7.) To get the Y-value for your max/min points, plug x into the ORIGINAL FUNCTION f(x).

Ex 1) Find intervals of incresing/decreasing and max/min pts of $f(x)$

$f(x) = x^4 - 32x + 4$

$f'(x) = 4x^3 - 32 = 0$ Diffence of cubes

$4(x^3 - 8) = 0$

$4(x-2)(x^2 + 2x + 4) = 0$

$x - 2 = 0$ $x^2 + 2x + 4 = 0$

$\boxed{x = 2}$ None → Quadratic Formula reveals no real solutions

$-$ $+$

$f'(0)$ 2 $f'(3)$ sign line

$f(x)$ is decreasing $(-\infty, 2)$ increasing $(2, \infty)$

min at $(2, f(2))$ or $(2, -44)$

Ex 2) Find intervals of inc/dec and max/min pts

$f(x) = x^3 - \frac{3}{2}x^2$

$f'(x) = 3x^2 - 3x = 0$

$3x(x-1) = 0$

$3x = 0$ $x - 1 = 0$

$\boxed{x = 0}$ $\boxed{x = 1}$

$+$ $-$ $+$

$f'(-1)$ 0 $f'(1/2)$ 1 $f'(2)$

inc $(-\infty, 0)$ and $(1, \infty)$

dec $(0, 1)$

max $(0, 0)$

min $(1, -1/2)$