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Theorems

Let f(x) be continuous on [a,b] and differen[tiable on] (a,b), then....

1. Mean Value Theorem (MVT): $f'(c) = \frac{f(b)-f(a)}{b-a}$
2. Extreme Value Theorem (EVT): f(x) has both [a min] and a max on the interval or at the endpoints [...] endpoints)
3. Intermediate Value Theorem (IVT): K is a nu[mber] between f(b) and f(a), then there is a c s[uch that] f(c) = K

Extrema and Concavity

$f(x)$	↗ ↘ ↗
$f'(x)$	+ − +

(inc/dec)(max/min) of f(x)

$f'(x)$	↗ ↘ ↗
$f''(x)$	+ − +

(concavity/inf pts) of f(x)

ex: to find max/min points and intervals of incr/decr using 1st derivative test

$f(x) = 4x^2 - 2x$

$f'(x) = 8x - 2 = 0$

$x = 1/4$

− | + ; $f'(0)$ 1/4 $f'(1)$

min point is $(1/4, f(1/4))$

decreasing $(-\infty, 1/4)$

increasing $(1/4, \infty)$

You may also use the 2nd derivative test $f''(x) = 8$, $f''(1/4) = +$ so min

find inflection points and intervals of concavity

$f(x) = x^3 - 3x^2$

$f'(x) = 3x^2 - 6x$

$f''(x) = 6x - 6 = 0$

$x = 1$

− | + ; $f''(0)$ 1 $f''(2)$

inflection pt: $(1, f(1))$

concave down: $(-\infty, 1)$

concave up: $(1, \infty)$

Miscellaneous – AP Exam tips

format:
- 30 MC no calculator (60 min)
- 15 MC calculator (45 min) } 50%
- 10 min break
- 2 FRQ calculator (30 min)
- 4 FRQ no calculator (60 min)

equation of tangent line $y - \_\_ = \_\_ (x - \_\_)$

(normal line) → perpendicular to tangent line

alternative form of derivative

$\lim_{x \to c} \frac{f(x) - f(c)}{x - c}$ = can use for absolute value

speed = |velocity|

increasing speed = velocity and acceleration have same sign

decreasing speed = velocity and acceleration have different sign

position = original

velocity = 1st derivative

acceleration = 2nd derivative

Limits

3 methods

Numerical	Graphical	Analytical Rules
y= tblstart: c Δtbl: .001	finger test lim exists if $x \to c$ both fingers come to same y-value	1. plug in, if 0/0 or #/0 – a. factor + cancel – b. rationalize + cancel – c. simplify fraction + cancel

special trig limit

a. $\lim_{x \to 0} \frac{\sin x}{x} = 1$

b. $\lim_{x \to 0} \frac{1 - \cos x}{x} = 0$

Infinite Limits

deg top > deg bottom = + or − ∞

deg top < deg bottom = 0

deg top = deg bottom = ratio of lead coefficient

Integration Rules

indefinite

$\int f'(x)dx = f(x) + C$

definite

$\int_a^b f'(x)dx = f(b) - f(a)$

u-sub shortcut

$\int f'(kx)dx = \frac{1}{k} f(kx) + C$

(k is a #)

derivative of integral FTOC

$f(x) = \int_3^{5x} 3t^2 - 4t \, dt$

$f'(x) = [3(5x)^2 - 4(5x)] \cdot 5$

riemman sums

x	0	5	10	15	20
f(x)	1	2	3	4	5

left (4 subs): (5)(1) + (5)(2) + (5)(3) + (5)(4)

right (4 subs): (5)(1) + (5)(3) + (5)(4) + (5)(5)

midpt (2 subs): (10)(2) + (10)(4)

trapezoid (4 subs): $\frac{5}{2}(1 + 2(2) + 2(3) + 2(4) + 5)$

Rev[...]
π[...]
Vol[...]

Con[...]
Cont[...]
1. lim[...]
2. f(c[...]
3. lim x→c[...]
ex–1s[...]
f(x) = [...]

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amos: Calc.forMulA.fOldAble

Related Rates
1. draw a picture
2. determine formula
3. write each variable in formula
4. write rate for each variable in formula wr
5. take derivative of formula wrt time
6. plug in what you know and solve for wha trying to find
7. place correct units

all else = units
area = $units^2$
volume = $units^3$

ex: $A = \pi r^2$

so $\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$? = 2(\pi)(2)(3)$

$\frac{dA}{dt} = 12\pi\ units^2$

$A =$ … $r = 2$ … $\frac{dA}{dt}$ … $\frac{dr}{dt}$

Differential Equations → 5pts on AP Exam
1. seperate variables (1)
2. integrate both sides (2) look for $\int \frac{u'}{u} = \ln|u| + C$
3. plug in initial condition (x,y) and solve for C (1)
4. place the C you found back into function and solve for y (1)

If given other variables like $\frac{dw}{dt} = t^3(100-w)$

change to $\frac{dy}{dx} = x^3(100-y)$

Revolved Volume
Revolved volume = place pencil on line you revolve about

$\pi \int_a^b (\text{outer function})^2 - (\text{inner function})^2 dx$ $\quad (R(x))^2 - (r(x))^2$

Volume of Cross Section: square $\int_a^b (\text{area})^2 d$

isosceles rt △ $\frac{1}{2} \int_a^b (\text{area})^2 dx$

semicircle $\frac{\pi}{8} \int_a^b (\text{area})^2$

rectangle $k \int_a^b (\text{area})$

Calculator Knowledge
derivative (graph or table) math 8 $\quad y_1 = f(x) \quad y_2 = \frac{d}{dx}[y_1]_{x=x}$

definite integration math 9 $\int_a^b [Y_1]\, dx$ integrate on home screen $\quad y_3 = \frac{d}{dx}[y_2]_{x=x}$

finding zeroes: put function in y_1

2nd, trace, 2, left bound, right bound, guess, enter, enter, enter

Finding intersection pts. (volumes of solids)

put functions in y_1 and y_2

2nd, trace, 5, 1st curve, 2nd curve, enter, enter, enter

to paste y_1: vars, y vars, function

Continuity
continuous at point c if
1. lim $\to c^-$ $x \to c^+$
2. f(c) exist
3. $\lim_{x \to c} = f(c)$ limit = functional value

ex – is f(x) continuous at x=2

$f(x) = \begin{cases} x^2-3, & x<2 \\ 2x-3, & x \geq 2 \end{cases}$

1. $\lim_{x \to 2^-} (2)^2 - 3 = 1$ $\quad \lim_{x \to 2^+} (2)(2) - 3 = 1$
2. $f(2) = 2(2) - 3 = 1$ $\quad \lim_{x \to 2} = f(2) = 1$

YES

Derivative Rules
Chain rule: $y = a(u)^n$
$y' = an(u)^{n-1} u'$

power rule: $y = ax^n$
$y' = anx^{n-1}$

product rule: $y = uv$
$y' = uv' + vu'$

quotient rule: $y = \frac{u}{v}$
$y' = \frac{vu' - [uv']}{(v)^2}$

Inverse trig
$y = \sin^{-1} u \quad y' = \frac{u'}{\sqrt{1-u^2}}$

$y = \tan^{-1} u \quad y' = \frac{u'}{1+u^2}$

$y = \csc^{-1} u \quad y' = \frac{-u'}{|u|\sqrt{u^2-1}}$

$y = \sec^{-1} u \quad y' = \frac{u'}{|u|\sqrt{u^2-1}}$

$y = \cos^{-1} u \quad y' = \frac{-u'}{\sqrt{1-u^2}}$

$y = \cot u \quad y' = \frac{-u'}{1+u^2}$

Inverse derivative
$g'(x) = \frac{1}{f'(g(x))}$

– g is the inverse function of f

Trig
$y = \sin u \quad y' = u'\cos u$

$y = \cos u \quad y' = -u'\sin u$

$y = \sec u \quad y' = u'\sec u \tan u$

$y = \csc u \quad y' = u'\csc u \cot u$

$y = \tan u \quad y' = u'\sec^2 u$

$y = \cot u \quad y' = -u'\csc^2 u$

Log and exponential
$y = e^u \quad y' = u'e^u$

$y = a^u \quad y' = u'a^u \ln(a)$

$y = \ln u \quad y' = \frac{u'}{u}$

$y = \log_a u \quad y' = \frac{u'}{u \ln(a)}$

Implicit differentiation
ex: $2xy - 3y^2 = 4y - 2x$

$2x\frac{dy}{dx} + 2y - 6y\frac{dy}{dx} = 4\frac{dy}{dx} - 2$

$\frac{dy}{dx} = \frac{-2-2y}{2x-6y-4}$ or $\frac{-1-y}{x-3y-2}$

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