

$$= \infty$$

NOTE The infinite series in Example 4 diverges very slowly. For instance, the sum of the first ten terms is approximately 1.6878196, whereas the sum of the first 100 terms is just slightly larger, 2.3250871. In fact, the sum of the first 10,000 terms is approximately 3.015021704. You can see that although the infinite series “adds up to infinity,” it does so very slowly!

EXERCISES FOR SECTION 8.3

In Exercises 1–10, use the Integral Test to determine the convergence or divergence of the series.

1. $\sum_{n=1}^{\infty} \frac{1}{n+1}$ ✓
2. $\sum_{n=1}^{\infty} \frac{2}{3n+5}$
3. $\sum_{n=1}^{\infty} e^{-n}$ ✓
4. $\sum_{n=1}^{\infty} ne^{-n/2}$
5. $\frac{1}{2} + \frac{1}{5} + \frac{1}{10} + \frac{1}{17} + \frac{1}{26} + \dots$ ✓
6. $\frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \frac{1}{9} + \frac{1}{11} + \dots$
7. $\frac{\ln 2}{2} + \frac{\ln 3}{3} + \frac{\ln 4}{4} + \frac{\ln 5}{5} + \frac{\ln 6}{6} + \dots$ ✓
8. $\frac{1}{4} + \frac{2}{7} + \frac{3}{12} + \dots + \frac{n}{n^2+3} + \dots$
9. $\sum_{n=1}^{\infty} \frac{n^{k-1}}{n^k+c}$, k is a positive integer ✓
10. $\sum_{n=1}^{\infty} n^k e^{-n}$, k is a positive integer

In Exercises 11 and 12, use the Integral Test to determine the convergence or divergence of the p -series.

11. $\sum_{n=1}^{\infty} \frac{1}{n^3}$ ✓
12. $\sum_{n=1}^{\infty} \frac{1}{n^{1/3}}$

In Exercises 13–20, use Theorem 8.11 to determine the convergence or divergence of the p -series.

13. $\sum_{n=1}^{\infty} \frac{1}{\sqrt[5]{n}}$ ✓
14. $\sum_{n=1}^{\infty} \frac{3}{n^{5/3}}$
15. $1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{4}} + \dots$ ✓
16. $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \dots$
17. $1 + \frac{1}{2\sqrt{2}} + \frac{1}{3\sqrt{3}} + \frac{1}{4\sqrt{4}} + \frac{1}{5\sqrt{5}} + \dots$ ✓
18. $1 + \frac{1}{\sqrt[3]{4}} + \frac{1}{\sqrt[3]{9}} + \frac{1}{\sqrt[3]{16}} + \frac{1}{\sqrt[3]{25}} + \dots$
19. $\sum_{n=1}^{\infty} \frac{1}{n^{1.04}}$ ✓
20. $\sum_{n=1}^{\infty} \frac{1}{n^{\pi}}$