UNIT 2

Limits and Continuity

Limits

3 methods

Numerical - Table (or calculator)

Graphical - What "y-value do you approach" as you approach a x-value from both sides?

Analytical - Rules (By hand)

steps

a.) Plug in c (result is limit)

b.) Indeterminate form $0/0$ or $\#/0$

i) Factor and cancel
ii) rationalize and cancel
iii) Simplify fraction

] Then plug in

A 2-sided limit exist if $\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x) = L$

"left" "right"

then

$\lim_{x \to c} f(x) = L$

"2 sided"

Ex 1

$$\lim_{x \to -1} \frac{3x^2 + 2x - 1}{1 + x}$$

$$\frac{3(-1)^2 + 2(-1) - 1}{1 + (-1)} = \frac{}{0}$$

Ind form so factor

$$\frac{(3x-1)\cancel{(x+1)}}{\cancel{(1+x)}}$$

so $\lim_{x \to -1} = (3(-1) - 1) = \boxed{-4}$

Ex 2) $\lim_{x \to 0} \frac{\sqrt{5+x} - \sqrt{5}}{x} = \frac{\sqrt{5+(0)} - \sqrt{5}}{0} = \frac{0}{0}$ ← Ind form

so rationalize $\frac{\sqrt{5+x} - \sqrt{5}}{x} \cdot \frac{\sqrt{5+x} + \sqrt{5}}{\sqrt{5+x} + \sqrt{5}} =$

$\lim_{x \to 0} \frac{5+x-5}{x(\sqrt{5+x} + \sqrt{5})} = \frac{x}{x(\sqrt{5+x} + \sqrt{5})} = \frac{1}{\sqrt{5+x} + \sqrt{5}}$

$\lim_{x \to 0} \frac{1}{\sqrt{5+x} + \sqrt{5}} = \frac{1}{\sqrt{5} + \sqrt{5}} = \boxed{\frac{1}{2\sqrt{5}}}$

Ex 3.) $\lim_{x \to -1} x^2 - x - 1 = (-1)^2 - (-1) - 1 = \boxed{1}$

Ex 4.) $\lim_{x \to 0} \frac{\frac{1}{2+x} - \frac{1}{2}}{x} = \frac{\frac{1}{2} - \frac{1}{2}}{0} = \frac{0}{0}$ ← Ind form

simplify fraction $\frac{\frac{2}{2}\frac{1}{2+x} - \frac{1}{2}\frac{2+x}{2+x}}{x} = \frac{\frac{2}{2(2+x)} \frac{-2-x}{2(2+x)}}{x}$

$\lim_{x \to 0} \frac{\frac{-x}{2(2+x)}}{x} = \frac{\frac{-x}{2(2+x)}}{\frac{x}{1}} = \frac{-x}{2(2+x)} \cdot \frac{1}{x} =$

$\frac{-x}{x \cdot 2(2+x)} = \frac{1}{2(2+x)}$ $\lim_{x \to 0}$ = plug in 0 $= \boxed{\frac{1}{4}}$

Continuity

complete together pg12

pg85 Texbook problem

a	$\lim\limits_{x \to a^+} f(x)$	$\lim\limits_{x \to a^-} f(x)$	$\lim\limits_{x \to a} f(x)$	$f(a)$	cont Y/N
0					
1					
1.5					
2					

A function $f(x)$ is continuous at a point $x=a$ if

1.) $\lim\limits_{x \to a^-} f(x) = \lim\limits_{x \to a^+} f(x) = L$

2.) $f(a)$ exist "No hole at a"

3.) $\lim\limits_{x \to a} f(x) = f(a) = L$ "#1 = #2"

Ex 1 $f(x) = \begin{cases} x^2 - 3, & x < 1 \\ 2x - 4, & x \geq 1 \end{cases}$ Is $f(x)$ continuous at $x=1$?

① $\lim\limits_{x \to 1^-} f(x) = (1)^2 - 3 = \boxed{-2}$ ① $\lim\limits_{x \to 1^+} f(x) = 2(1) - 4 = \boxed{-2}$

② $f(1) = 2(1) - 4 = \boxed{-2}$ ③ $\lim\limits_{x \to 1} f(x) = f(1) = \boxed{2}$

$\boxed{\text{YES}}$

Ex 2)

$$f(x) = \begin{cases} -3 & , x \le 0 \\ x^2 & ; 0 < x < 2 \\ x+2 & , x \ge 2 \end{cases}$$

Is f(x) continuous at x = 0?

① $\lim_{x \to 0^-} f(x) = -3$ ① $\lim_{x \to 0^+} f(x) = (0)^2 = 0$ $\lim_{x \to 0} f(x) =$ Does Not Exist

condition 1 fails so f(x) is not continuous at x=0

Is f(x) continuous at x = 4?

① $\lim_{x \to 4^-} f(x) = (4)+2 = \boxed{6}$ ① $\lim_{x \to 4^+} f(x) = (4)+2 = \boxed{6}$

② $f(4) = (4)+2 = \boxed{6}$

③ $\lim_{x \to 4} f(x) = f(4) = \boxed{6}$

f(x) is cont at x=4
YES

Infinite Limits

Degree Top < Degree Bttm = 0

Deg Top = Deg Bttm = ratio of lead coefficient a/b

Deg Top > Deg Bttm = + or − ∞ use calculator table.

Same rules as horizontal Asymptotes

Ex1) $\lim_{x \to -\infty} \frac{ax^2+bx+c}{d-Kx^2}$ $a,b,c,d,K \in {}^{+}\mathbb{Z}$

Deg Top = Deg bttm = 2 → Answer $a/{-K}$

Ex2) $\lim_{x \to \infty} \frac{4x^2-3x}{2x^5-5x+4}$

Answer 0

Deg Top < Deg Bttm

Ex3)

$\lim_{x \to -\infty} \frac{4x^3-2x}{x-5}$

Use calculator table

Deg Top > Deg Bttm → Answer $+\infty$

How to find a Limit in the Calculator

Steps 1) Type function in Y_1

2.) Go to table set
start table at $\underline{\ c\ }$
Δ Tbl at $\underline{.001}$

3.) Go to table and look at what Y_1 closes down on and up on. Use the arrows to scroll.

Ex 1) $\lim\limits_{x \to -1} \dfrac{3x^2+2x-1}{1+x}$ $y_1 = (3x^2+2x-1)/(1+x)$

Tbl start = $\underline{-1}$
Δ Tbl = $\underline{.001}$

$\boxed{\text{Answer } -4}$

-1.002	-4.006
-1.001	-4.003
-1	Error
-.999	-3.997
-.998	-3.994

Y values close down and up on -4

AP FRQ Reference for Unit 2

2011 6a
2011 Form b 2a
2008 6d
2003 6a
2012 4c

You can research more on your own. Note most FRQ involve continuity. Check practice MC exams for Limits questions