UNIT 1

Exponent Rules

$x^a \cdot x^b = x^{a+b}$

$(x^a)^b = x^{ab}$

$\frac{x^a}{x^b} = x^{a-b}$

$x^{-a} = \frac{1}{x^a}$

$x^0 = 1$

$\left(\frac{x}{y}\right)^a = \frac{x^a}{y^a}$

$\sqrt[n]{x^p} = x^{p/n}$

$\sqrt{\frac{x}{y}} = \frac{\sqrt{x}}{\sqrt{y}}$

$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$

$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \cdot \frac{d}{c}$

$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$

Examples

① $\sqrt{x} \cdot \sqrt[3]{x} = x^{1/2} \cdot x^{1/3} = x^{\frac{1}{2}+\frac{1}{3}} = \boxed{x^{5/6}}$

② $\left(\frac{x^{-3}}{x^5}\right)^4 = (x^{-8})^4 = \boxed{x^{-32}}$

Calculus

Write each expression as a single line expression with no variables in the denominator, (negative exponents are ok)

Ex) $\frac{x^3(x^2+x^5)}{3x^3} = \frac{x^2+x^5}{3} = \boxed{\frac{1}{3}x^2 + \frac{1}{3}x^5}$

Ex) $\frac{x^{-2}(4x^3-5)}{x^5} = x^{-7}(4x^3-5) = \boxed{4x^{-4} - 5x^{-7}}$

Ex) $\frac{\sqrt{x}(\sqrt[4]{x})}{5x} = \frac{x^{1/2}(x^{1/4})}{5x} = \frac{x^{3/4}}{5x} = \boxed{\frac{1}{5}x^{-1/4}}$

Expressing 1

Ways of writing 1

$x^0 = 1$

$5-4=1$

$\frac{4}{4}=1$

$-6+7=1$

one $=1$

$\sqrt{1}=1$

1 is "Anything into itself."

$\frac{5x}{5x}=1$ $\frac{2y}{2y}=1$

$\frac{12}{12}=1$ $\frac{\sqrt{3}}{\sqrt{3}}=1$

We use 1 in

$\boxed{\text{Arithmetic}}$ $\boxed{\frac{5}{5}} \cdot \frac{1}{3} + \frac{2}{5} \cdot \boxed{\frac{3}{3}} = \frac{5}{15} + \frac{6}{15} = \frac{11}{15}$

$\boxed{\text{Algebra}}$

$$\frac{2}{\sqrt{3}} \cdot \boxed{\frac{\sqrt{3}}{\sqrt{3}}} = \frac{2\sqrt{3}}{3}$$

$\boxed{\text{Geometry - Changing Degrees to radians and vice versa}}$

$$\frac{\pi}{4} \cdot \boxed{\frac{180}{\pi}} = \frac{180\pi}{4\pi} = 45°$$

$\boxed{\text{Algebra 2}}$

$$\frac{2}{4+\sqrt{5}} \cdot \frac{4-\sqrt{5}}{4-\sqrt{5}} = \frac{8-2\sqrt{5}}{11} \text{ or } \frac{8}{11} - \frac{2\sqrt{5}}{11}$$

Recall that by multiplying something by 1 doesn't change the "value" but may change the appearance.

Riemann Sums

Purpose - Approximate area between a function (curve) and the x-axis by the use of rectangles and trapezoids.

2 methods

Table

x	a	b	c	d	e
f(x)	f(a)	f(b)	f(c)	f(d)	f(e)

Function

$$\int_a^b \underbrace{f'(x)}_{\text{height}} \underbrace{dx}_{\text{base}}$$

difference between x's

4 types

- left
- right
- midpoint
- trapezoid

Ex) Approximate the area using all 3 rectangle methods of equal base of 10

x	0	5	10	15	20	25	30
f(x)	2	4	1	6	3	5	-2

L

(10) (2) = 20
(10) (1) = 10
(10) (3) = 30

$\boxed{60}$

R

(10) (1) = 10
(10) (3) = 30
(10) (-2) = -20

$\boxed{20}$

M

(10) (4) = 40
(10) (6) = 60
(10) (5) = 50

$\boxed{150}$

Ex: Approximate the area using all 3 rectangle methods. Use $0 \le x \le 6$ with 3 equal bases

$$\int_0^6 x^2 \, dx$$

$L = \boxed{40}$

$(2)(0) = 0$
$(2)(4) = 8$
$(2)(16) = \underline{32\ +}$

$R = \boxed{112}$

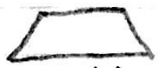
$(2)(4) = 8$
$(2)(16) = 32$
$(2)(36) = \underline{72\ +}$

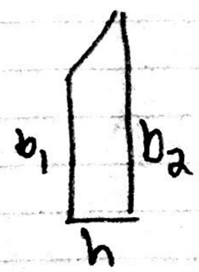
$M = \boxed{70}$

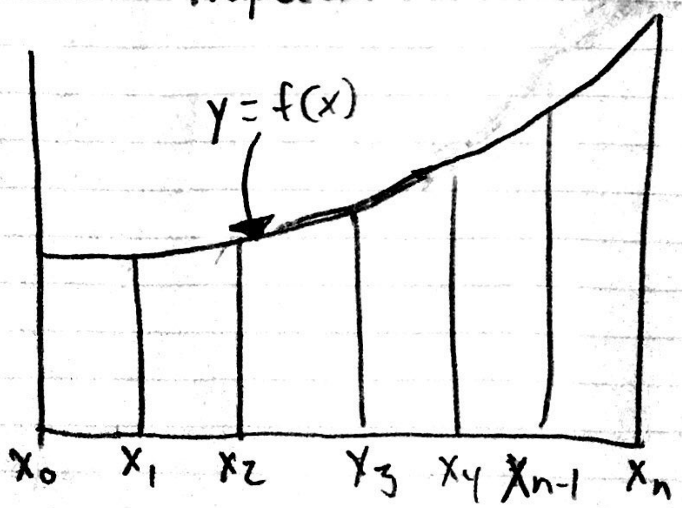
$(2)(1) = 2$
$(2)(9) = 18$
$(2)(25) = \underline{50\ +}$

Trapezoid Rule – A trapezoid is a triangle with 2 bases.

so  triangle $A = \frac{hb}{2}$

so  trapezoid $A = \frac{h}{2}(b_1 + b_2)$





$n-1$ = next to last term

n = last term

Sum

$$\frac{h}{2}\left[f(x_0)+f(x_1)\right]+\frac{h}{2}\left[f(x_1)+f(x_2)\right]+\frac{h}{2}\left[f(x_2)+f(x_3)\right]+$$

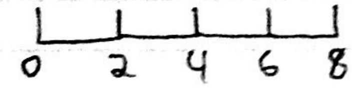
$$\frac{h}{2}\left[f(x_3)+f(x_4)\right]+\frac{h}{2}\left[f(x_4)+f(x_{n-1})\right]+\frac{h}{2}\left[f(x_{n-1})+f(x_n)\right]$$

A <u>shortcut</u> - can only be used with trapezoids of equal heights

$$\frac{h}{2}\left[f(x_0)+2f(x_1)+2f(x_2)+2f(x_3)+2f(x_4)+2f(x_{n-1})+f(x_n)\right]$$

<u>Ex</u> - Use a trapezoid R.Sum with 4 <u>equal</u> subintervals to approximate the area

$$\int_0^8 x^2+1\,dx$$

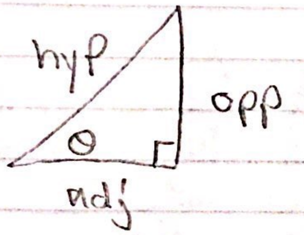


$$\frac{2}{2}(1+5)+\frac{2}{2}(5+17)+\frac{2}{2}(17+37)+\frac{2}{2}(37+65)$$

$$6 + 22 + 54 + 102 = \boxed{184}$$

You try shortcut

Trig Review

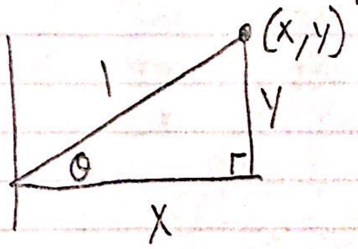


$\sin\theta = \frac{O}{H}$ $\quad\quad$ $\csc\theta = \frac{H}{O}$

$\cos\theta = \frac{A}{H}$ $\quad\quad$ $\sec\theta = \frac{H}{A}$

$\tan\theta = \frac{O}{A}$ $\quad\quad$ $\cot\theta = \frac{A}{O}$

unit circle - radius/hyp = 1



* Must Know

$\sin\theta = y$ $\quad\quad$ $\csc\theta =$ Flip y

$\cos\theta = x$ $\quad\quad$ $\sec\theta =$ Flip x

$\tan\theta = y/x$ $\quad\quad$ $\cot\theta = x/y$

Arc = Inverse = $f^{-1}(x)$ = "The angle whose"

Ex 1) Find $\tan\left(\frac{5\pi}{6}\right)$, since tan is y/x at $\frac{5\pi}{6}$ we have $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

refer to $\frac{\pi}{6}$

$$\frac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = \frac{1}{2} \cdot \frac{-2}{\sqrt{3}} = \boxed{\frac{-1}{\sqrt{3}} \text{ or } -\frac{\sqrt{3}}{3}}$$

Ex 2) Find the $\sec\left(7\pi/4\right)$

refer to $\pi/4$

Since sec is flip x we have
at $7\pi/6$ $\left(\sqrt{2}/2, -\sqrt{2}/2\right)$ we flip x

$$\boxed{\frac{2}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}}$$

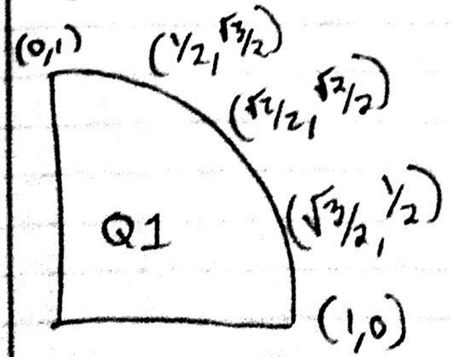
Ex 3) Find $\arctan(\sqrt{3})$ or $\tan^{-1}(\sqrt{3})$
this means "the angle whose" tan is $\sqrt{3}$
since tan is y/x we get $\left(1/2, \sqrt{3}/2\right)$ and $\left(-1/2, -\sqrt{3}/2\right)$

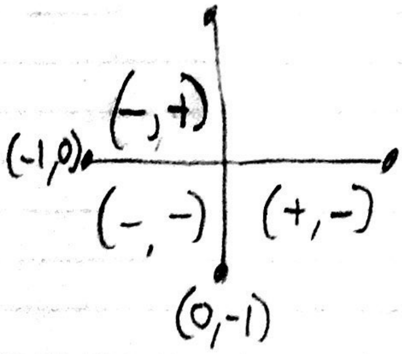
$\frac{\pi}{3}$ $\frac{4\pi}{3}$

so $\dfrac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \frac{\sqrt{3}}{2} \cdot \frac{2}{1} = \sqrt{3}$ verified

$$\boxed{\therefore \arctan(\sqrt{3}) = \frac{\pi}{3} \text{ and } \frac{4\pi}{3}}$$

If you learn quadrant 1 and the 4 poles you may use symmetry for the others





AP FRQ Reference for Unit 1

AB

2017	1a
2016	1b
2015	3b
2014	4c
2013	3c
2012	1c
2011	2b
2010	2b

2011 Form B 5b

2010 Form B 3a

You can research, study, practice, and grade yourself on years 1998-2009