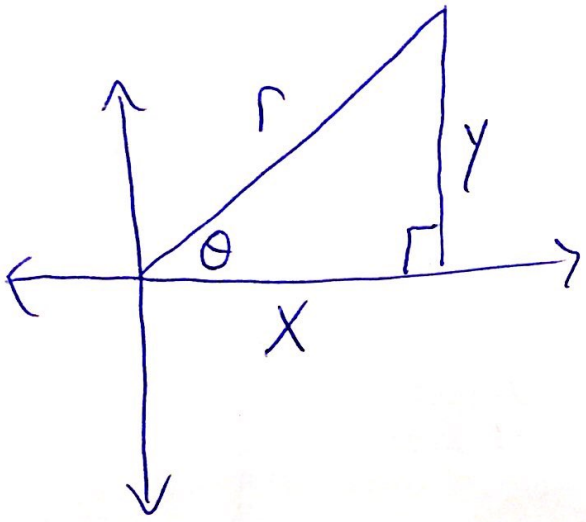
Polar Equations



$x^2 + y^2 = r^2$

$\tan\theta = \frac{y}{x}$

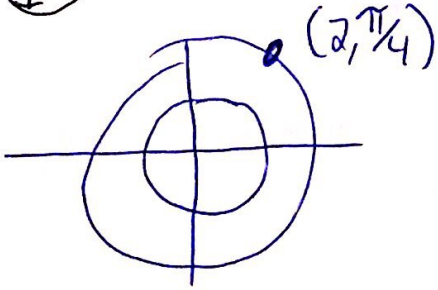
$\sin\theta = \frac{y}{r}$ so $y = r\sin\theta$

$\cos\theta = \frac{x}{r}$ so $x = r\cos\theta$

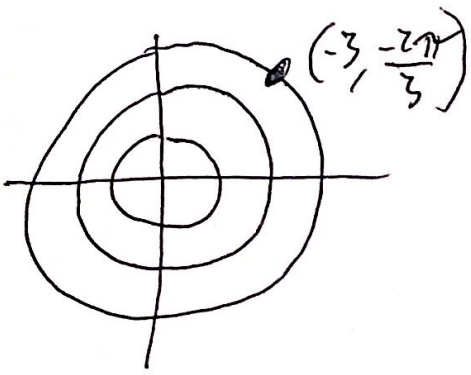
Plot polar points

a,) angle 1st
radius 2nd
if r is negative go to opposite pole

Ex 1 $(2, \pi/4)$



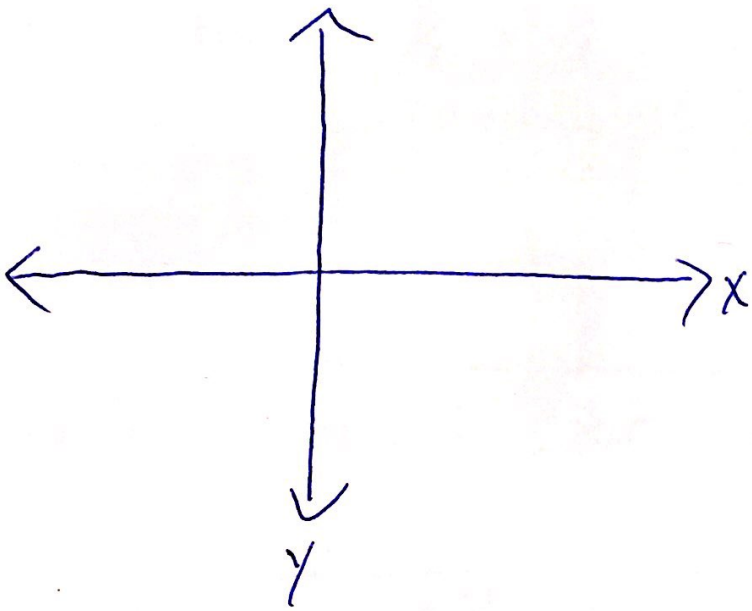
Ex 2 $(-3, \frac{-2\pi}{3})$



* Note $(-3, -2\pi/3)$ is same as * $(\pi/3, 3)$

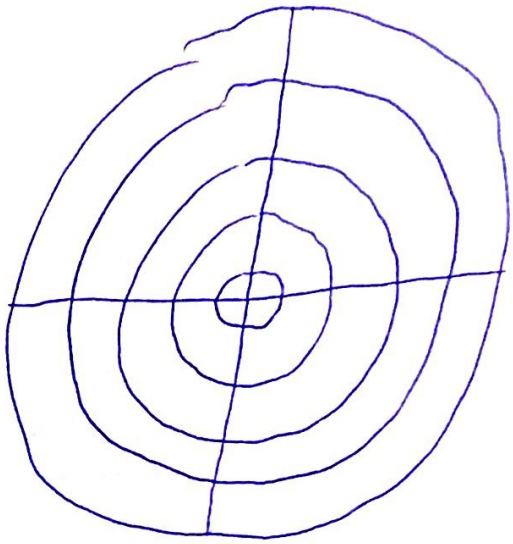
Polar Equations

Rectangular Graph



(x, y)

polar graph



(r, θ)

radius $= r$ angle $= \theta$

I. Convert rectangular point to polar point

$(x, y) \longrightarrow (r, \theta)$

use $x^2 + y^2 = r^2$

so $r = \sqrt{x^2 + y^2}$

$\tan\theta = y/x$

so $\theta = \tan^{-1}(y/x)$

Ex 1 Convert to polar $(1,1)$

$r = \sqrt{x^2 + y^2}$ $\quad r = \sqrt{1+1} = \sqrt{2}$

$\tan\theta = \frac{y}{x} = \frac{1}{1} = 1$ so $\tan^{-1}(1) = \frac{\pi}{4}$

$\boxed{(\sqrt{2}, \frac{\pi}{4})}$

Ex 2 Convert to polar $(0,-5)$

$r = \sqrt{(0)^2 + (-5)^2} = 5$

$\tan\theta = \frac{-5}{0}$ $\quad \tan^{-1}\left(\frac{-5}{0}\right)$ "Angle whose tangent is undefined $\frac{}{0}$ at $\frac{\pi}{2}, \frac{3\pi}{2}$

so $\theta = \frac{3\pi}{2}$

$\boxed{(5, \frac{3\pi}{2})}$

II Convert polar to rectangular

use $x = r\cos\theta$, $y = r\sin\theta$

Ex 1) Convert to rectangular $\left(4, \frac{2\pi}{3}\right)$

$x = r\cos\theta$ so $x = 4\cos\left(2\pi/3\right)$

$4\left(-\frac{1}{2}\right) = -2$

$y = r\sin\theta$ so $y = 4\sin\left(2\pi/3\right)$

$4\left(\sqrt{3}/2\right) = 2\sqrt{3}$

$\left(-2, 2\sqrt{3}\right)$

Ex 2.) Convert to rectangular $\left(1, 7\pi/4\right)$

$x = 1\cos\left(7\pi/4\right) = +\frac{\sqrt{2}}{2}$

$y = 1\sin\left(7\pi/4\right) = -\frac{\sqrt{2}}{2}$

$\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$